Formal appreciation of art

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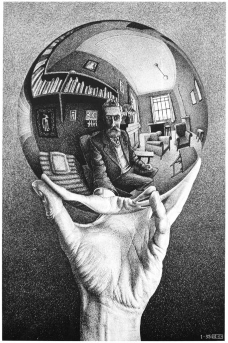
June 26, 2022

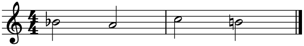
Objective

Capture multimodal subjective preference with $\kappa : \mathbb{R}^n \to \mathbb{R}$

$\overset{?}{>}_\kappa$



$\overset{?}{>}_\kappa$



And ever changing, like a joyless eye

That finds no object worth its constancy

$\overset{?}{>}_\kappa$

κ

Related work: Formal Theory of Creativity

Among comparable observations the subjectively most beautiful is the one with the shortest description (Schmidhuber 2012)

$$B(D \mid O, t) = -C(p(t), D)$$
$$I(D \mid O, t) = \frac{\partial B(D \mid O, t)}{\partial t}$$

Beauty is the negative number of bits needed to encode D with observer's model $p(t)$. Interestingness is the derivative of beauty.

Background: λ-calculus

Descriptions are equivalent to λ-terms, where a λ-term is either a:

- Variable a, where the symbol a is drawn from infinite alphabet
- Application (ab), if a and b are λ-terms
- Abstraction $(\lambda a.b)$, if a is a variable and b is a λ-term

$$
\begin{aligned}
1 &\longleftrightarrow \lambda sz.sz \\
2 &\longleftrightarrow \lambda sz.s(sz) \\
+ &\longleftrightarrow \lambda mnsz.ms(nsz)
\end{aligned}
$$

Figure: Encoding into λ-term

$$
\begin{aligned}
((\lambda a.b)c) &\longrightarrow_{\beta} [a \rightarrow c]b \\
(+\,1\,2) &\longrightarrow_{\beta *} 3
\end{aligned}
$$

Figure: Reduction of λ-term

Background: λ-calculus

$$(+\,1\,2) \longrightarrow_{I\beta} ((\lambda x.(+\,1\,x))\,2)$$
$$1 + 2 \longrightarrow_{I\beta} f(2) \text{ where } f(x) = 1 + x$$

Figure: Abstraction of λ-term

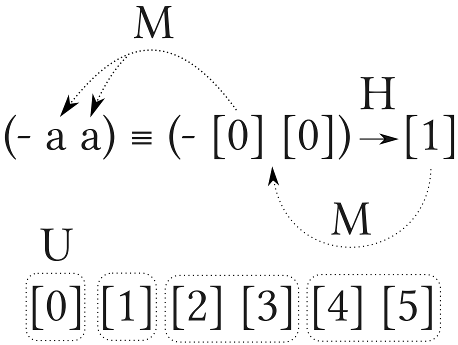
$$|a| = 1 \quad \text{if } a \text{ is a variable or constant}$$
$$|(ab)| = |a| + |b|$$
$$|(\lambda a.b)| = |b|$$

Figure: Length of λ-term

Background: Equivalence graph

E-graph is a tuple (U, M, H) relating e-classes (sets of e-nodes)

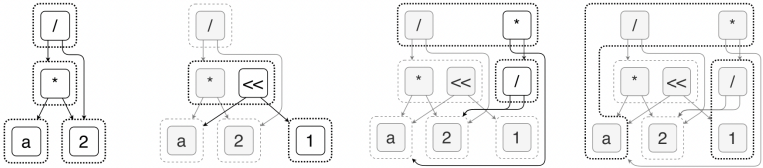
- U – disjoint-set (union-find) over e-class ids
- M – mapping between e-class ids and e-classes
- H – mapping between e-nodes and e-class ids



Background: Equality saturation

1. Saturation: With a set of rewrite rules $\Lambda = \{\lambda_\ell \longleftrightarrow \lambda_r\}$ and given a term x find all equivalent terms x^Λ modulo Λ, by applying all rules perpetually until fixed-point
2. Extraction: Among x^Λ pick the most optimal term x_*^Λ

Example of $(a \times 2)\,/\,2 \longrightarrow a$



(a) Initial e-graph contains $(a \times 2)/2$.

(b) After applying rewrite $x \times 2 \to x \ll 1$.

(c) After applying rewrite $(x \times y)/z \to x \times (y/z)$.

(d) After applying rewrites $x/x \to 1$ and $1 \times x \to x$. [1]

[1] (Willsey et al. 2021)

Semi-Formal view

Saturation: making of descriptions

$$x \longrightarrow x_0^\Lambda \xrightarrow{\Lambda} x^\Lambda \longrightarrow x_*^\Lambda$$

Abstraction: improvement of descriptions

$$\Lambda, x^\Lambda \xrightarrow{\lambda} \Lambda_+, x^{\Lambda_+}$$

1. By virtue of making sense of observations, one has to acquire certain abstractions
2. Observations are as good as useful are abstractions present in their descriptions
3. All possible descriptions are taken into account, in all contexts, discounted by some prior
4. Goodness of an abstraction is measured as far as it helps in simplifying observer's history of observations

Formal view

1. Observer's aim: $\min_{\Lambda} |H_*^\Lambda|$ or $\min_{H,\Lambda} |H_*^\Lambda|$ (Friston 2010)
2. Observer's tool: $\Lambda = \{\lambda\} = \{\lambda_\ell \longleftrightarrow \lambda_r\}$
3. Observer's observations: $x \subset H = \ldots 1101111010101101$
4. Observer's descriptions: $x^\Lambda = \{x_m^\Lambda\} = \{\lambda_n \mid \lambda_n \longleftrightarrow x_0^\Lambda\}$
5. Observer's refactorings: $\lambda(x^\Lambda) = \{\lambda \mid \lambda \in I\beta^*(x_m^\Lambda)\}$
6. Abstraction's utility:

$$\kappa(\lambda \mid x_m^\Lambda, \Lambda) = 2^{|x_m^{\Lambda-\lambda}| - |x_m^{\Lambda+\lambda}| - |\lambda|}$$

$$\kappa(\lambda \mid x, \Lambda) = \sum_m^{|x^\Lambda|} 2^{|x_*^\Lambda| - |x_m^\Lambda|} \kappa(\lambda \mid x_m^\Lambda, \Lambda)$$

7. Observation's utility:

$$\kappa(x \mid H, \Lambda) = \sum_{\lambda \in \lambda(x^\Lambda)} \frac{\kappa(\lambda \mid x, \Lambda)}{|\lambda(x^\Lambda)|} \kappa(\lambda \mid H, \Lambda)$$

Example: Number-list language

Table: Primitives from the number-list language

name	description	λ-term
0	zero	$(\lambda sz.z)$
S	successor of natural number	$(\lambda nsz.(s((ns)z)))$
ø	empty list	$(\lambda x.x)$
.	list constructor	$(\lambda htf.((fh)t))$

$$x = [1, 2, 3]$$
$$x_0^\Lambda = (.\ (S\ 0)\ (.\ (S\ (S\ 0)\ (.\ (S\ (S\ (S\ 0)))\ ø)))$$

Figure: Example of an observation

Example: Abstractions

$$x_0^\Lambda = (.\ (S\,0)\ (.\ (S\,(S\,0))\ (.\ (S\,(S\,(S\,0)))\ ø)))$$

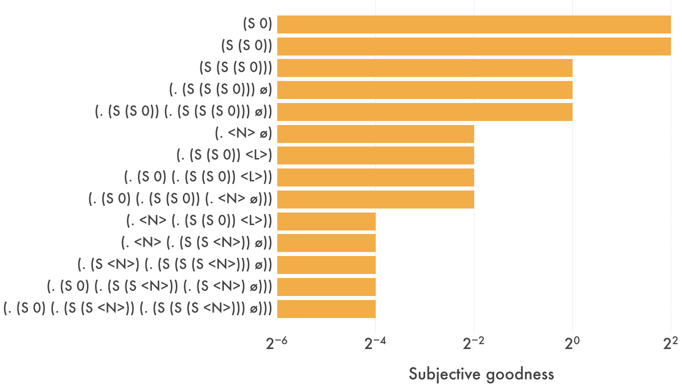


Figure: Relative goodness of abstractions from $\lambda(x_0^\Lambda)$

Example: Saturated observation

$$\text{starts-with-1-and-2} \longleftrightarrow (\lambda x.(.\ (S\ 0)\ (.\ (S\ (S\ 0)\ x)))$$
$$\text{ends-with-3} \longleftrightarrow (.\ (S\ (S\ (S\ 0))\ \text{ø})$$

Figure: Selected abstractions

$$x_1^\Lambda = (\text{starts-with-1-and-2}\ (.\ (S\ (S\ (S\ 0))\ \text{ø}))$$
$$x_2^\Lambda = (.\ (S\ 0)\ (.\ (S\ (S\ 0))\ \text{ends-with-3})$$
$$x_*^\Lambda = x_3^\Lambda = (\text{starts-with-1-and-2}\ \ \text{ends-with-3})$$

Figure: Example of a saturated observation with selected abstractions

Example: Scale language

Table: Primitives from the scales language

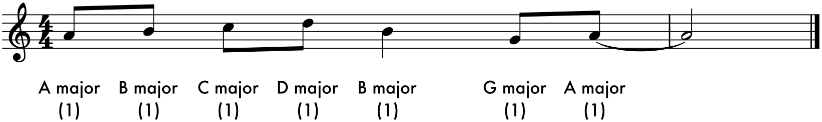
$\mathbb{N}$	natural numbers (Church numerals)
minor, diminished, etc.	scales
scale[$\mathbb{N}$]	indexing of the scale
$\mathbb{N}$ + scale[$\mathbb{N}$]	shift key of the scale (e.g. D♭ minor)
$\uparrow, \downarrow$	next, previous index (+1, -1)
loop $\mathbb{N} \uparrow, \downarrow$	repeat $\uparrow, \downarrow$ $\mathbb{N}$ times

1. D♭ chromatic[1] $\longleftrightarrow$ D♭ minor[1] $\longleftrightarrow$ C locrian[2] $\longleftrightarrow \ldots$
2. D♭ minor[1], D♭ minor[2] $\longleftrightarrow$ D♭ minor[1, 2]
3. D♭ minor[1, 2] $\longleftrightarrow$ D♭ minor[1, $\uparrow$]
4. D♭ minor[$\uparrow$] $\longleftrightarrow$ D♭ minor[loop 1 $\uparrow$]
5. D♭ minor[loop $\mathbb{N} \uparrow, \uparrow$] $\longleftrightarrow$ D♭ minor[loop $\mathbb{N} + 1 \uparrow$]

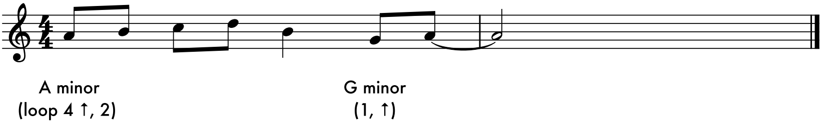
Figure: Abstractions for the scales language

Example: Jazz licks descriptions

The Lick

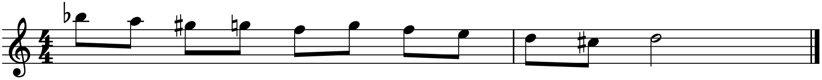


The Lick



Example: Jazz licks descriptions

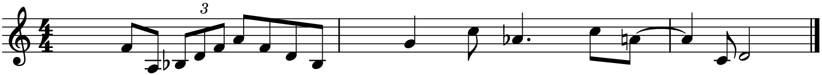
Barry Harris



B♭ chromatic
(loop 4 ↓)

F bebop-major
(1, ↑, loop 4 ↓, ↑)

Charlie Parker #1

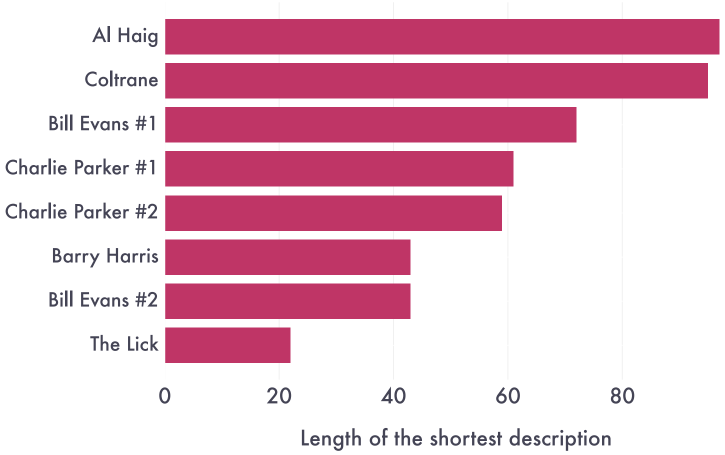


B♭ major-chord
(3, loop 5 ↑, loop 3 ↓)

A♭ major-chord
(4, 2, ↓)

C pentatonic
(1, ↓, loop 2 ↑)

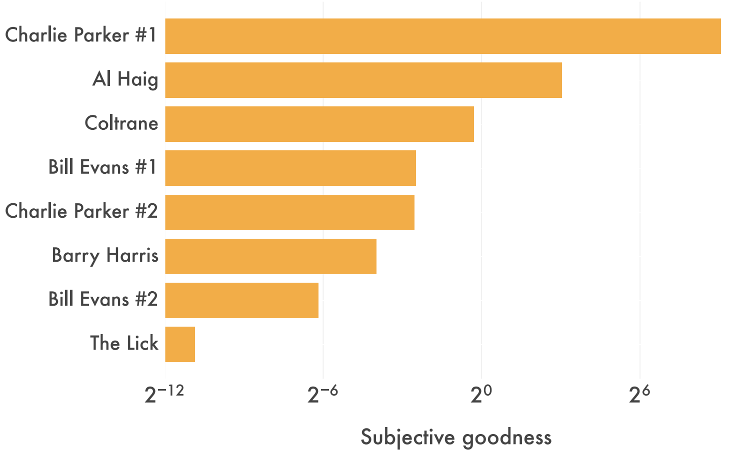
Example: Jazz licks descriptions



Example: Comparison

Set H in $\kappa(x \mid H, \Lambda)$ to the concatenation of all licks

$$\kappa(x \mid H, \Lambda) \neq -|x_*^\Lambda|$$



Critique

1. "λ-calculus is an overkill, it's usage is vague"
    - λ-calculus is much more commonplace nowadays than alternatives
    - Both major works we rely on (Willsey et al. 2021; Ellis et al. 2021) use it ubiquitously
    - 3-page proof of completeness and consistency of inverse beta-reduction (Ellis et al. 2021)
2. "The selection of scales in the language needs to be argued more" / "why not consider other type of chords"
    - It is most certainly not a complete language, but a proof of concept
3. "The resulting most useful abstractions are neither presented nor explained."
    - They are both presented and explained in the case of number-list language
    - In the case of scales language their presentation would require too much explaination

Contribution

1. Python e-graph implementation
    - ~500 LOC, however no optimizations
    - Alternative: Quiche (EGRAPHS'22), also no optimizations
    - Alternative: python FFI to egg (Willsey et al. 2021)
2. Musical analysis on e-graphs
    - Original, however not complete
    - Almost complete with (Nandi et al. 2021)
3. Multimodal subjective comparison
    - Purely conceptual
    - Lacking similarity

References

Ellis, Kevin et al. (2021). "DreamCoder: Bootstrapping Inductive Program Synthesis with Wake-Sleep Library Learning". In: *Proceedings of the 42nd ACM SIGPLAN International Conference on Programming Language Design and Implementation*. New York, NY, USA: Association for Computing Machinery, pp. 835–850. ISBN: 9781450383912. URL: https://doi.org/10.1145/3453483.3454080.

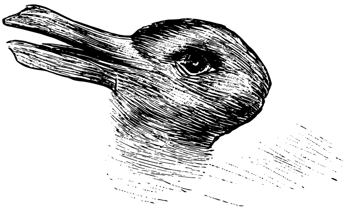
Friston, Karl (2010). "The free-energy principle: a unified brain theory?" In: *Nature reviews neuroscience* 11.2, pp. 127–138.

Nandi, Chandrakana et al. (Oct. 2021). "Rewrite Rule Inference Using Equality Saturation". In: *Proc. ACM Program. Lang.* 5.OOPSLA. DOI: 10.1145/3485496. URL: https://doi.org/10.1145/3485496.

Schmidhuber, Jürgen (2012). "A formal theory of creativity to model the creation of art". In: *Computers and creativity*. Springer, pp. 323–337.

Willsey, Max et al. (2021). "Egg: Fast and Extensible Equality Saturation". In: *Proc. ACM Program. Lang.* 5.POPL. DOI: 10.1145/3434304. URL: https://doi.org/10.1145/3434304.

Afterword



Kaninchen und Ente.

1. Rabbit
2. Duck
3. Duck-Rabbit (a picture which is two things: rabbit & duck)
4. Textbook example of tricks in perception
5. Wittgenstein's duck-rabbit
6. Picture associated with Wittgenstein, even though he attributed it to Jastrow, who had taken it from Harper's Weekly issue of November 19th 1892, whose editors had taken it …